

Physics 4261: Lectures for week 1

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1.1 Introduction: atoms

The topic of the course is “atomic physics” and hence the fundamental unit we study will be the atom. The most common question I get when I tell people I study atomic physics (aside from people who confuse the terms “atomic” and “nuclear”), is “don’t atoms make up everything?” And, in a way, they do. But atoms, as we will study them in this course, are actually pretty rare, at least on earth (they may be more common in astrophysics). The reason is that we want to consider atoms *in isolation*. Atoms in solids and liquids are very close to together, and their behavior is very different from isolated atoms, hence the need for condensed matter physics. Even in the gas phase, the most common gases (O_2 , N_2) are actually molecules, and not atoms. So what we are going to study is in fact something of a rarity, which occurs most commonly in vapor cells built especially for the purpose. Nonetheless, we will see that, compared to our colleagues who study more practical things, we can describe atoms using relatively simple models and capture the physics with shocking accuracy. For example, atomic clocks are at 18 digits these days.

1.2 Atomic physics today

To give a little perspective, and to motivate our studies, I will outline what I see as the th main areas in atomic physics today:

- **“Quantum Simulation”** This is my own area of research and consists of trying to arrange atoms in such a way as to simulate other quantum systems, typically condensed matter systems, but also nuclear or astrophysical systems. This is motivated by the observation (by Feynman) that only a quantum mechanical system (as atoms are) can efficiently model another quantum system.
- **“Quantum Information”** A related idea, seeking instead to harness the quantum behavior of atoms for general purpose computing (i.e. factoring large primes), or encrypting data for storage and communication.
- **“Precision Measurement”** This includes atomic clocks, and efforts to measure symmetry breaking, or weak effects like gravity. Many theories predict symmetry breaking involving high energy particles (e.g. electroweak theory). These effects should show up either at high energy, in collider experiments (as many have), or as very weak effects in atoms. However, considering the huge cost of colliders, and the years of measuring needed to get even a few digits, precision atomic measurements are an exciting and complementary alternative.

- **“Basic Atomic Properties”** Most of the basic properties of common atoms have been measured to extraordinary precision already, but there are some properties still to be measured, like rare isotope masses, isotopic ratios, and hyperfine structure.

1.3 Pre-Schrödinger equation atomic physics

Just to give brief context, as this is not a quantum mechanics or history of quantum mechanics course.

- **Ultraviolet catastrophe** From statistical mechanics, we expect $k_B T$ of energy per radiation mode. But there are infinitely many modes going to higher and higher frequency. Planck corrected this by relating the frequency to the energy with the constant $h = 2\pi * \hbar$, and considering discrete quanta of light we now call “photons”. He arrived at the formula for the density of states

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^2} \frac{1}{e^{\hbar\omega/k_B T} - 1}.$$

- **Photoelectric effect** Classically, one expects a stronger light source to produce a higher electrical field at a metal surface, and to therefore rip off more electrons and make them go faster. However, only the number of electrons depends on intensity, the emitted kinetic energy T is given by

$$T = h\nu - \phi,$$

where h is Planck’s constant from above and ν is the frequency of the light, ϕ is the work function (depends on the metal chosen), below which no emission occurs.

- **Rutherford scattering** When energetic, charged α particles are incident on gold foil, most of the atoms passed straight through or had a small deflection. This is expected if the charge is in small clumps (the nuclei), but not if the charge is spread out (somewhat hilariously called “plum pudding”).
- **Atomic emission** When atom vapor is heated, it emits light not over a continuous spectrum, but instead at discrete frequencies. Equating frequency of the light to energy (as above), we conclude that the atom has discrete energy states. However, the classical orbital model of the atom predicts any energy is possible. In fact, it is unbounded below, which is another problem!

1.4 Bohr model (Foot 1.2-1.3)

The observed emission frequencies occur at wavenumbers

$$\tilde{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{n'^2} \right),$$

for integers n, n' . Note the book’s use of wavenumbers, or inverse wavelengths, which are really units of frequency or energy (just multiply by c or hc). Also note alternative terminology, the $n = 1$

series is called the Lyman series, the $n = 2$ the Balmer series, the $n = 3$ the Paschen, and there are even higher ones. Within each series, the $n \rightarrow n + 1$ line is called α , the $n \rightarrow n + 2$ line β , and so forth.

These observations imply discrete levels with energy $\propto 1/n^2$. Let us see how we might find these. Let us assume an electron of mass $m = m_e = 9.11 \times 10^{-31}$ kg and (negative) charge $e = 1.60 \times 10^{-19}$ C orbiting a nucleus of mass $M = M_p = 1.67 \times 10^{-27}$ kg and the same but positive charge.

By classical arguments, let's assume we have a circular orbit. Setting the force inward to the centripetal force for circular motion

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{p^2}{mr}.$$

By the virial theorem, the binding energy of this orbit is

$$E_b = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{2r}.$$

We can eliminate the unknown factor r if we knew E_b , but we don't and we still get an infinite number of possible orbits. Let us then consider that we might quantize the orbit according to the deBroglie wavelength. The deBroglie wavelength will be $\lambda_{dB} = h/p$. We assume that each orbit is an integer number of deBroglie wavelengths, so that

$$\begin{aligned} 2\pi r &= nh/p \\ p &= n\hbar/r, \end{aligned}$$

which leads to the final result

$$\begin{aligned} \frac{1}{r} &= \frac{e^2}{4\pi\epsilon_0} \frac{m}{n^2\hbar^2}, \\ E_b &= -\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m}{2n^2\hbar^2}. \end{aligned}$$

In real units, this binding energy works out to 13.6 eV, or 2.2×10^{-18} J. If we quantize radiation by $E = h\nu$, this leads to $\nu = 3.3 \times 10^{15}$ Hz. Using $E = hc\lambda^{-1}$, this gives $\lambda^{-1} = 1.1 \times 10^7$ m⁻¹. The book uses the valid (but rare) SI unit m⁻¹, but the literature more often uses cm⁻¹.

1.5 Fine-structure constant and Bohr radius

A common way of writing the Bohr energy levels is to use the fine structure constant. This dimensionless ratio shows up a lot in atomic physics. Taking $n = 1$, and setting $-E_b = mv^2/2$ (virial theorem), we obtain

$$\alpha = \frac{v}{c} = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx 1/137.$$

In this convention, $E_b = -mc^2\alpha^2/(2n^2)$. We also define a Bohr radius a_0 by

$$-2E_b = mc^2\alpha^2/n^2 = \frac{e^2}{4\pi\epsilon_0 a_0} = \frac{\alpha\hbar c}{a_0},$$

$$a_0 = \frac{\hbar}{mc\alpha},$$

taking $n = 1$ in the last step.

1.6 X-rays (Foot 1.5)

This section will mostly be to understand the scaling for nuclei with large charge. If the charge of the nucleus is instead Ze , with Z the atomic number, we can go back to the previous expression, and replace

$$\frac{e^2}{4\pi\epsilon_0} \quad \text{by} \quad \frac{Ze^2}{4\pi\epsilon_0}.$$

Then we see that the frequencies are all multiplied by Z^2 , leading to the famous $Z \sim f^{1/2}$ result of H. G. J. Moseley. We also introduce the X-ray terminology, where the $n = 1$ series is called K , the $n = 2$ series L , and so on. Note that we will have, later on, an entirely different letter-to-number scheme for angular momentum, i.e. $s, p, d, f, g, h \dots$. Atomic physics is fun, just be glad we're not doing molecules!

1.7 Reduced mass and nuclear recoil

Here we show a trick to take into account the recoil of the nucleus (i.e. the nucleus does not have infinite mass). Let us write the classical Hamiltonian as

$$H = \frac{p_e^2}{2m} + \frac{p_N^2}{2M} + \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{r}_e - \vec{r}_p|}.$$

First we do a little trick to eliminate the center of mass,

$$H = \frac{1}{2} \frac{(p_e + p_N)^2}{m + M} + \frac{1}{2} \frac{m + M}{mM} \left(\frac{Mp_e - mp_N}{m + M} \right)^2 + \frac{e^2}{4\pi\epsilon_0} \frac{1}{|r_e - r_p|},$$

$$H = \frac{p_T^2}{2(m + M)} + \frac{p_\Delta^2}{2\mu} + \frac{e^2}{4\pi\epsilon_0} \frac{1}{|r_\Delta|},$$

with $\mu = mM/(m + M)$ (reduced mass). Therefore, we will henceforth consider a single particle system with mass given by the reduced mass. Note that this remains valid even in a fully quantum mechanical treatment.

1.8 Zeeman effect (Foot 1.8)

A good sketch of the Zeeman apparatus is shown on page 17 of your book (section 1.8.1). I'm going to walk you through the apparatus to show you how it works.

The conclusion from the data is that each line splits into three peaks upon application of a magnetic field, with the center peak at π polarization, and the side peaks at σ polarization. Let's now work through a (totally classical) explanation. Zeeman assumed that the electron was bound in a harmonic oscillator potential, governed by the equation

$$m\dot{\vec{v}} = -k\vec{r} - e\vec{v} \times \vec{B}.$$

Simplifying a little bit and taking \vec{B} along \hat{z} gives,

$$\ddot{\vec{r}} + \frac{eB}{m}\dot{\vec{r}} \times \hat{z} + \frac{k}{m}\vec{r} = 0.$$

Define Ω_C as the cyclotron resonance eB/m (twice the Larmor resonance from the book), and $\omega_0^2 = k/m$ as the oscillator frequency,

$$\ddot{\vec{r}} + \Omega_C\dot{\vec{r}} \times \hat{z} + \omega_0^2\vec{r} = 0.$$

We propose a trial solution of the form

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} e^{-i\omega t}.$$

This leads to a matrix equation

$$\begin{pmatrix} \omega_0^2 - \omega^2 & -i\omega\Omega_C & 0 \\ i\omega\Omega_C & \omega_0^2 - \omega^2 & 0 \\ 0 & 0 & \omega_0^2 - \omega^2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0.$$

We choose ω to make the determinant zero. Note that the off-diagonal terms proportional to ω make this slightly different from an ordinary matrix diagonalization. Obviously $\omega^2 = \omega_0^2$ is a solution. The other solutions are found by solving

$$(\omega_0^2 - \omega^2)^2 = \omega^2\Omega_C^2.$$

The solutions are

$$\begin{aligned} \omega^2 &= \omega_0^2 + \frac{\Omega_C^2}{2} \pm \sqrt{\omega_0^2\Omega_C^2 + \Omega_C^4}, \\ \omega^2 &\approx \omega_0^2 \pm \omega_0\Omega_C, \\ \omega &\approx \omega_0 \pm \Omega_C/2. \end{aligned}$$

The approximations are based on $\Omega_C \ll \omega_0$, which for typical optical transitions and real-world magnetic fields will be the case. Thus, from this splitting Zeeman was able to conclude that atoms contained particles with the same charge-to-mass ratio as electrons from cathode ray experiments.